Modeling of Hovering Type AUV Test Bed with Four Thrusters and Two Hulls Design

Eko Henfri Binugroho [1], Young Bong Seo [2], and Jae Weon Choi [3]
[1] Electronics Engineering Polytechnic Institute of Surabaya
[2] Innovation Center for Engineering Education, Pusan National University
[3] School of Mechanical Engineering, Pusan National University
E-mail: sragen@eepis-its.edu

Abstract

This paper proposes a design of hovering Type Autonomous Underwater Vehicle (AUV) using four thruster and two hulls. The state vector representation of AUV kinematics and the mathematical modeling of their dynamics of motion are presented. Several assumptions are used to simplify the design since the effect of that parameter is small enough thus can be neglected in the modeling process. DC motor is used as the actuator of the thruster. The thrusters used in the system are designed to have the same motor power and propeller characteristic.

Keywords: *Modelling, Hovering-type, AUV, Thusters, Hulls.*

1. Introduction

The Autonomous Underwater Vehicle (AUV) is designed to overcome the limitation of Remotely Operated Underwater Vehicles (ROV), since it is untethered, fully automated submersible platforms capable of performing underwater tasks and missions with their onboard sensor, navigation and payload equipment. Despite its advantages over an ROV, its design process is far more complex with respect to its programming especially in controller and navigation part which demands and autonomy operation.

Today, the Autonomous Underwater Vehicle (AUV) industry is growing dramatically with the increase in the reliability and technical abilities of these vehicles. The goal in underwater robotics at present is to create fully self-contained, intelligent, decision-making AUVs [1]. In order to accomplish this goal, many researches are being carried out worldwide with particular emphasis on autonomy, navigation, object detection, energy sources and information systems.

One of the basic platform of the AUV is the mechanical design of the AUV it self. Several type of AUV are designed. The most popular type which is developed by many researchers is torpedo type that has the advantage of the small power usage because of its small number of actuator. But the torpedo type also has the limitation about its maneuverability. The other design of the AUV is hovering type that relatively consume more power than torpedo type AUV since it has more actuator. In the apposite condition it has the advantage in the maneuverability part since it has more DOF can be controlled.

This paper presents a design of hovering type AUV using four thrusters and two hulls. The DOF that can be actively controlled by the thrusters are the movement are surge, heave, and yaw movement only. The roll movement is designed to be passively controlled by the design of the AUV's Hulls properties and their displacement.

2. The AUV Modelling

Determining the state vector representation and the modeling of their motion is the first step to control this AUV.

2.1 AUV Kinematics

To analyze the motion of the AUV in 6 degree of freedom (DOF), it is convenient to represent its coordinate into two frames, body-fixed and earth frame coordinate that depicted at Fig .1.

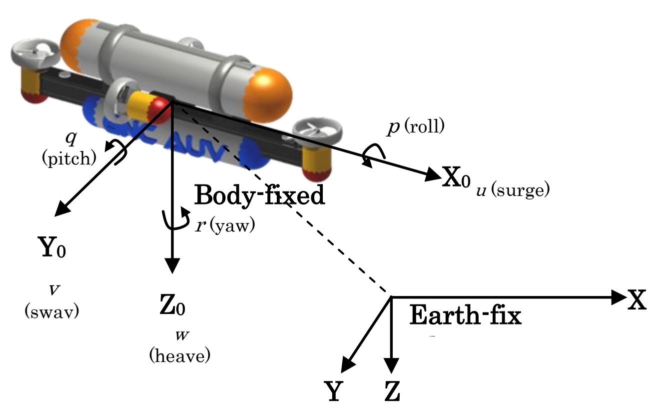

Fig. 1. Body-fixed and earth-fixed reference frames.

AUV body axes X_0, Y_0, and Z_0 coincide with the axes with the principal axes of inertia. Position and orientation of the AUV are suggested to be described relative to inertial reference frame *XYZ*, while the linear and angular velocities to be expressed in the body-fixed coordinate system [2].

The AUV motion in 6 DOF are described by vector

$$\begin{aligned} P &= \left[P_1^T, P_2^T\right]^T; \quad P_1 = [x, y, z]^T; \quad P_2 = [\phi, \theta, \psi]^T, \\ V &= \left[V_1^T, V_2^T\right]^T; \quad V_1 = [u, v, w]^T; \quad V_2 = [p, q, r]^T, \\ \tau &= \left[\tau_1^T, \tau_2^T\right]^T; \quad \tau_1 = [X, Y, Z]^T; \quad \tau_2 = [K, M, N]^T. \end{aligned} \tag{1}$$

P denotes the position and orientation vector of AUV with coordinates in the earth-fixed frame, while *V* denotes linear and angular velocity state vectors of the vehicle with coordinates in the body-fixed frame and τ is forces and moments acting to the vehicle in the body-fixed frame.
When converting from body to world coordinates, it is conventional in robotics that the first rotation be ψ about the *z* axis, followed by θ about the intermediate y axis, and lastly ϕ about the second intermediate *x* axis. In general, an AUV should be restricted to the following rotation angles,

$$\begin{aligned} &-\pi < \phi \le \pi, \\ &-\pi/2 < \theta < \pi/2, \\ &0 \le \psi < 2\pi. \end{aligned} \tag{2}$$

The rotation matrix of frame {B} relative to frame {W} which is the rotation matrix for converting from body-fixed to earth-fixed coordinates can be written as

$${}^W_B R(P_2) = R_z(\psi) R_y(\theta) R_x(\phi). \tag{3}$$

where

$$R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, \tag{5}$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}. \tag{6}$$

2.2 Velocity Transformation

The linear velocities of the AUV in earth-fixed coordinate can be represented as,

$$\dot{P}_1 = {}^W_B R(P_2) V_1 = R V_1. \tag{7}$$

The relation of body-fixed angular velocity vector $V_2 = [p, q, r]^T$ and the Euler angle rate vector $\dot{P}_2 = \left[\dot{\phi}, \dot{\theta}, \dot{\psi}\right]^T$ can be expressed as

$$\dot{P}_2 = {}^W_B W V_2, \tag{8}$$

where

$${}^W_B W = W = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \tag{9}$$

2.3 AUV Dynamic

The dynamic model presented in this section is based on the underwater robotic models proposed by Fossen [2] and Yuh [3]. AUV dynamic model is derived from Newton-Euler motion equation given by

$$M\dot{V} + C(V)V + D(V)V + G(P_2) = \tau, \tag{10}$$

where:

M	= mass and inertia matrix including added mass,
$C(V)$	= coriolis and centripetal terms matrix including added mass,
$D(V)$	= hydrodynamic damping matrix,
$G(P_2)$	= gravitational and buoyancy vector,
τ	= external force and torque input vector,
V	= velocity state vector.

M consists of both a rigid body mass and inertia M_{RB} and a hydrodynamic added mass M_A, which can be written as

$$M = M_{RB} + M_A. \tag{11}$$

The Rigid body mass M_{RB} is calculated based on the frame $\{B\}$ that is located at the vehicle's center of gravity, $r_G = [0 \quad 0 \quad 0]^T$. The parameters of the added mass matrix M_A are dependent on the shape of the AUV. However they are constants when the vehicle is fully submerged. The parameters are usually in the vicinity of 10% to 100% of the corresponding parameters in the rigid body mass matrix.

The hydrodynamic damping matrix *D*(*V*) represents the drag and lift forces acting on a moving underwater vehicle that will be highly non linear and coupled if vehicle moves at high speed. These drag forces can be separated into two different terms consisting of a linear damping D_L and quadratic damping D_Q which can be written as,

$$D(V) = D_L + D_Q|V| \tag{12}$$

where

$$\begin{aligned} D_L &= diag\left\{X_u, Y_v, Z_w, K_p, M_q, N_r\right\}, \\ D_Q &= diag\left\{X_{u|u|}, Y_{v|v|}, Z_{w|w|}, K_{p|p|}, M_{q|q|}, N_{r|r|}\right\}. \end{aligned} \tag{13}$$

The gravitational and buoyancy vector, $G(P_2)$ is defined as

$$G(P_2) = \begin{bmatrix} (W-B)\sin\theta \\ -(W-B)\cos\theta\sin\phi \\ -(W-B)\cos\theta\cos\phi \\ B\cos\theta\left(y_B\cos\phi - z_B\sin\phi\right) \\ -B\left(z_B\sin\theta + x_B\cos\theta\cos\phi\right) \\ B\left(x_B\cos\theta\sin\phi + y_B\sin\theta\right) \end{bmatrix}. \tag{14}$$

The external force and torque vector produced by the thrusters is defined as

$$\tau = \left[X, Y, Z, K, M, N\right]^T = LU. \tag{15}$$

with

$$U = \left[T_1, T_2, \cdots, T_n\right]^T. \tag{16}$$

where L is a mapping matrix and U is a thrust vector. U is the vector of thrusts produced by the vehicle's thrusters. The number of thrust values in U depends on the number of thrusters on the vehicle. The mapping matrix L is essentially a 6 × n matrix that uses U to find the overall forces and moments acting on the vehicle.

3. Dynamic Model of the Designed AUV

3.1 The Physical Design of the AUV

The AUV is designed with four thrusters and two hulls as depicted in Fig. 2 and 3.

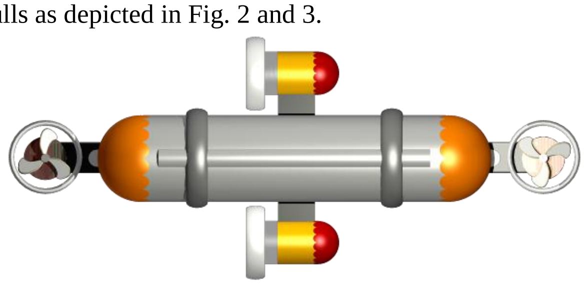

Fig.2 The top view of the designed AUV

Two thrusters T_1, T_2 are used to control the AUV in the surge movement in the apposite direction, thus the AUV can move forward and backward. The other two thrusters T_3, T_4 are used in the heave movements for diving and moving to the surface. These thrusters also maintain the pitch angle of the AUV in its movement.

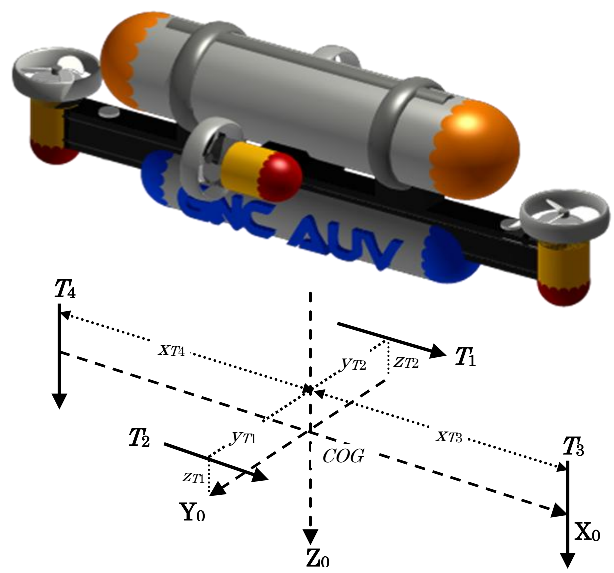

Fig.3 Designed AUV frame reference and Torque vector

3.2 Assumptions that Taken for the Modeling

The simplification of the model was required because if all parameters are used it will quite complex thus it will be very time consuming and difficult process. In order to simplify the AUV dynamic, several assertions below are needed.

- The vehicle travels at low speeds (less than 2m/s).
- Roll movement is passively controlled by AUV design properties and considered to be negligible.
- The vehicle is considered to be symmetrical about its three planes.

The justification for the decoupling of the degrees of freedom is based on the fact that:

- The vehicle is fairly symmetrical about its three planes,
- The off-diagonal elements of the dynamic model matrices are much smaller than their counterparts,
- The hydrodynamic damping coupling is negligible when the vehicle travels at low speed [4].

If the decoupling is valid thus the coriolis and centripetal terms matrices become negligible and consequently can be eliminated from the dynamic model. The dynamic model in Eq. 10 can be simplified as

$$M\dot{V} + D(V)V + G(P_2) = \tau \tag{17}$$

3.3 Simplifying the Dynamic Model Matrices

With the vehicle fixed-body frame is positioned at the center of gravity and since the vehicle is assumed fairly symmetrical about all axes, then AUV rigid-body mass M_{RB} in the Eq. 11 can be simplified to a good approximation to,

$$M_{RB} = diag\{m, m, m, I_x, I_y, I_z\} \tag{18}$$

Furthermore, since roll and pitch dynamic is negligible then Eq. 18 can be wrtitten as

$$M_{RB} = diag\left\{m, 0, m, 0, I_y, I_z\right\}. \tag{19}$$

with m is the mass of the AUV, I_y and I_z is the inertial force in y and z axis respectively.

Since the vehicle travels at low speed, thus added mass from (4.21) can be simplified as

$$M_A = diag\left\{X_{\dot{u}}, 0, Z_{\dot{w}}, 0, M_{\dot{q}}, N_{\dot{r}}\right\}, \tag{20}$$

with $X_{\dot{u}}$, $Z_{\dot{w}}$, $M_{\dot{q}}$, and $N_{\dot{r}}$ are added mass in the surge, heave, pitch and yaw movement respectively.

The hydro dynamic damping $D(V)V$ from Eq. 12 also can be simplified as

$$D(\boldsymbol{v}) = diag\left\{X_u + X_{u|u|}|u|,\ 0, Z_w + Z_{w|w|}|w|,\ 0, M_q + M_{q|q|}|q|,\ N_r + N_{r|r|}|r|\right\} \tag{21}$$

For a rigid-body moving trough an ideal fluid, the hydrodynamic damping matrix will be real, non-symmetrical and strictly positive.

The center of gravity is located at $r_G = [0,0,0]^T$, while the center of buoyancy r_B was found to be $r_B = [0,0,z_B]^T$ for a good approximation, since z_B is not aligned to the center of the gravity in z axis. With roll dynamic is negligible with $\phi = 0$, then the gravitational force from Eq. 14 can be simplified as

$$G(P_2) = \begin{bmatrix} (W-B)\sin\theta \\ 0 \\ -(W-B)\cos\theta \\ 0 \\ -z_B B\sin\theta \\ 0 \end{bmatrix}. \tag{22}$$

Based on the external force and torque vector produced by the thrusters from the designed AUV that depicted in Fig. 3, the forces and moments acting to the vehicle in the body-fixed frame can be written as

$$\tau = LU = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ -z_{T1} & -z_{T2} & -x_{T3} & x_{T4} \\ y_{T1} & -y_{T2} & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}. \tag{23}$$

3.4 State Space Model of the AUV Dynamic

By inserting Eq. 19-22 into Eq.17, the complete AUV can be obtained. By neglecting the sway and roll dynamic then the dimension of the state space model of this dynamic can be reduced that yields

$$\begin{bmatrix} \frac{-(X_u + X_{u|u|}|u|)}{m + X_{\dot{u}}} & 0 & 0 & 0 \\ 0 & \frac{-(Z_w + Z_{w|w|}|w|)}{m + Z_{\dot{w}}} & 0 & 0 \\ 0 & 0 & \frac{-(M_q + M_{q|q|}|q|)}{I_y + M_{\dot{q}}} & 0 \\ 0 & 0 & 0 & \frac{-(N_r + N_{r|r|}|r|)}{I_z + N_{\dot{r}}} \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ r \end{bmatrix} + \begin{bmatrix} \frac{-(W-B)\sin\theta}{m + X_{\dot{u}}} \\ \frac{(W-B)\cos\theta}{m + Z_{\dot{w}}} \\ \frac{z_B B \sin\theta}{I_y + M_{\dot{q}}} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{m + X_{\dot{u}}} & \frac{1}{m + X_{\dot{u}}} & 0 & 0 \\ 0 & 0 & \frac{-1}{m + Z_{\dot{w}}} & \frac{-1}{m + Z_{\dot{w}}} \\ \frac{-z_{T1}}{I_y + M_{\dot{q}}} & \frac{-z_{T2}}{I_y + M_{\dot{q}}} & \frac{-x_{T3}}{I_y + M_{\dot{q}}} & \frac{x_{T4}}{I_y + M_{\dot{q}}} \\ \frac{z_{T1}}{I_z + N_{\dot{r}}} & \frac{-z_{T2}}{I_z + N_{\dot{r}}} & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{r} \end{bmatrix} \tag{24}$$

Finally, linear and angular velocity on the earth-fixed coordinate that can be integrated directly to obtain position and orientation of the AUV can be calculated from the transformation of its body-fixed velocity can be written as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\theta\cos\psi & \sin\theta\cos\psi & 0 & 0\\ \cos\theta\sin\psi & \sin\theta\sin\psi & 0 & 0\\ -\sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 0 & \tan\theta\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1/\cos\theta\end{bmatrix}\begin{bmatrix}u\\ w\\ q\\ r\end{bmatrix}. \quad (25)$$

4. Control Input and Thruster Model

The propeller used in the AUV design is screw propeller and the model used is bilinear thruster model.

4.1 Bilinear Thruster Model

A 1st order approximation of the open trust T and torque Q for single screw propeller can be derived from lift force calculation [28]. The non-dimensional thrust coefficient K_T and torque coefficient K_Q coefficients can be described as

$$K_T(J_0)=\frac{T}{\rho D^4 n|n|},\ \ K_Q(J_0)=\frac{Q}{\rho D^5 n|n|} \quad (26)$$

where D is the propeller diameter, ρ is thee water density, n is the propeller shaft speed, and

$$J_0=\frac{u_a}{nD} \quad (27)$$

is the advance speed ratio with u_a is the ambient water velocity at the propeller (speed of the water going into propeller in m/s).

K_T and K_Q usually show a liner behavior in J_0 and can be approximated by

$$K_T=\alpha_1 J0+\alpha_2, \qquad K_Q=\beta_1 J0+\beta_2, \quad (28)$$

where $\alpha_1,\alpha_2,\beta_1,\text{and}\,\beta_2$ are non-dimensional constants of the thruster force. Eq. (4.49) implies that mathematical expression of Q and T ca be written as

$$T=T_{n|n|}|n|n-T_{n|u_a|}|n|u_a \quad (29)$$

$$Q=Q_{n|n|}|n|n-Q_{n|u_a|}|n|u_a \quad (30)$$

$T_{n|n|}$, $T_{n|u_a|}$ are positive propeller coefficients given by the propeller characteristics that depend on the propeller diameter, shape of the duct, water density, etc. These coefficients will also depend on n and u_a since Eq. 29 and Eq. 30 are only first order approximation to more general expression. However, experiments have shown that this dependency can be neglected for practical conditions of operation [2].

In many practical applications the bilinear thruster model can be approximated by affine model. The propeller force in multivariable model can be simplified as

$$\tau=B_1 u \quad (31)$$

where B_1 is matrices of appropriate dimensions with new control variable, and u is the speed of the AUV in apposite direction of the thruster force. Thus the propeller force in the i-th DOF developed by j-th propeller can be written as

$$\tau_i=B_{ij}u_j; \qquad B_{ij}=T_{|n|n} \quad (32)$$

4.2 Optimal Distribution of Control Forces

For Underwater vehicle where the input matrix B is non-square and number of thruster is bigger or equal to number of controllable DOF, it is possible to find an optimal distribution control of energy [5]. Considering the quadratic energy function

$$J=\frac{1}{2}u^T W u \quad (33)$$

which can be minimized subject to $\tau-Bu=0$.
Considering the Lagrangian

$$L(u,\lambda)=\frac{1}{2}u^T W u+\lambda^T(\tau-Bu) \quad (34)$$

where λ denotes the Lagrange multiplier, W is the selected positive weighting matrix. By differentiating Lagrangian L with respect to u, and put $\tau=Bu$, then τ can be calculated as

$$\tau=BW^{-1}B^T\lambda. \quad (35)$$

Assuming that $BW^{-1}B^T$ is non-singular, then the optimal solution for the Lagrange multiplication can be found as

$$\lambda=\left(BW^{-1}B^T\right)^{-1}\tau. \quad (36)$$

Suggest that u can be calculated as $u=B_W^{\dagger}\tau$, the generalized inverse $B_W^{\dagger}$ can be calculated as

$$B_W^{\dagger}=W^{-1}B^T\left(BW^{-1}B^T\right)^{-1}. \quad (37)$$

Since state space dimension of the designed AUV can be reduced by removing sway and roll dynamics as shown in Eq. 24, then B matrix can be reduced as well, resulting the number thruster equal to the number of state which and makes B matrix become a square matrix. If all inputs are equally weighted as $W=I$, thus generalized inverse $B_W^{\dagger}$ in Eq. 37 can be simplified as

$$B^{\dagger} = B^T\left(BB^T\right)^{-1}. \tag{38}$$

Using Eq. 38, since the B matrix is square then $B^{\dagger}$ is simply equal to B^{-1}, thus control input u can be calculated as

$$u = B^{-1}\tau . \tag{39}$$

4.3 Actuator Dynamic

Thruster force produced by a propeller can be controlled by controlling actuator rotation speed. For small AUV application, most thruster systems are driven by small DC motors. The state space dynamic model of speed-controlled DC motor including the propeller dynamic can be written as

$$\begin{bmatrix} \dot{i}_a \\ \dot{n} \end{bmatrix} = \begin{bmatrix} \dfrac{-R_a}{L_a} & \dfrac{-2\pi K_m}{L_a} \\ \dfrac{K_M}{2\pi J_m} & \dfrac{Q}{2\pi J_m} \end{bmatrix} \begin{bmatrix} i_a \\ n \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_a , \tag{40}$$

where

R_a = armature resistance,
L_a = armature inductance,
u_a = armature voltage (motor input voltage),
i_a = armature current,
K_m = motor torque constant,
J_m = moment inertia of rotor and propeller,
Q = load torque of the propeller defined in Eq. 30
n = motor rotation (rotation/s).

Finally by inserting the value of n of the DC motor to Eq. 29, the thruster force for each thruster can be calculated.

5. Conclusions

In this paper the model of the AUV using hovering type-model with four thrusters and two hulls design is presented. The model presented in this paper use several assumptions based on the AUV physical design that designed to work an area with very low environmental disturbance.

1. Simplification of the AUV model based on the AUV mechanical design is needed, because several parameters just give a small effect to the system, thus can be neglected from the model.
2. The designed AUV is fairly symmetrical about its three planes, and it designed to travel at low speed, the coupling effect of each DOF can be neglected, thus the mass matrix can be calculated from its diagonal properties only.
3. Since the disturbances from the water environment is very low, then there is no thruster acting in y axis thus sway dynamic can be neglected.
4. The roll dynamic can assumed to be zero, since the mechanical properties using two hulls design can reject this movement.

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