

Analysis of Output Current Ripple of Three-phase PWM Inverter under Discontinuous Modulation Techniques

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Abstract-In this paper, an analysis of output current ripple of three-phase PWM inverters with discontinuous modulation is presented. The analytical expressions for discontinuous modulation injection signal are derived. It is followed by derivation of rms value of output current ripple of three-phase PWM inverter as a function of modulation index. Analysis of the influence of injection signal phase shift of the rms output current ripple three-phase PWM inverter is given. It is shown that at the same switching frequency with sinusoidal PWM, the discontinuous PWM produces lower rms output current ripple at high modulation index. A 30° lag phase shift results in lower rms output current ripple. Phase shifting from symmetrical discontinuous PWM gives lower modulation index. Simulation results are included to verify the derived expressions.

Keywords: *Discontinuous modulation, Ripple, Phase shift, Modulation index.*

1. Introduction

Voltage source inverters are widely used in motor drives, active filters, UPS (Uninterrupted Power Supply). One of switching inverter control technique is the technique of pulse width modulation or better known as the PWM technique. This technique compares two signals, called the reference signal, typically a sinusoidal signal with a carrier signal, usually a triangular signal having a much higher frequency. Signal comparison results are used for switching the inverter. Until now, the PWM technique being developed to search for switching signal that could reduce the current ripple.

It is known in the PWM technique that an arbitrary signal can be injected into the sinusoidal reference signal. This signal has various shapes and is intended to reduce the ripple current. One of the injection signals that are developed is discontinuous injection signal. The injected signal is made so as to produce a discontinuous reference signal for 60° in the period when the voltage reference signal reaches its maximum. Until now known there are several kinds of discontinuous modulation signals. Which will be discussed in this thesis is a discontinuous symmetrical and asymmetrical PWM.

There are already quite a lot of researches on discontinuous PWM in recent years. The analysis using discontinuous symmetrical modulation signal on the input side of the five-phase PWM inverter is performed [3]. Other study has been done on the analysis of symmetrical discontinuous modulation signals on the output side of the five-phase PWM inverter [4]. Despite a lot of research in this field, as far as the authors know, no work has shown the influence of phase shift on the output current ripple of three-phase PWM inverter.

In this paper, an analysis of output current ripple of three-phase PWM inverters with discontinuous modulation is presented. The analytical expressions for discontinuous modulation injection signal are derived. Based on these expressions, rms value of output current ripple of three-phase PWM inverter as a function of modulation index is derived. In addition, an analysis of the influence of injection signal phase shift of the rms output current ripple three-phase PWM inverter is given. It is shown that at the same switching frequency with sinusoidal PWM, the discontinuous PWM produces lower rms output current ripple at high modulation index. A 30° lag phase shift results in lower rms output current ripple. Also, Phase shifting from symmetrical discontinuous PWM gives lower modulation index.

To verify the derived expressions, simulation with three-phase inverter is conducted. Good correlations between the simulation and calculated results are obtained.

2. Analysis of Output Current Ripple

This section will discuss the current ripple of three-phase PWM inverters with discontinuous PWM. The analysis will be conducted on a single switching wave period. For the sake of simplicity, the load represented with a delta connection and each phase of it consists of resistance R_l , inductance L_l , and a sinusoidal counter emf in series connection. Three-phase PWM inverter scheme shown in figure 1. Figure 2 shows the three-phase sinusoidal modulation signal. From Figure 2 we can take one switching period "A" for analysis. Signal waveforms one switching period can be seen in Figure 3. To perform this analysis, It is assumed that the inverter supply comes from the ripple-free DC voltage source. In addition, dead

time effects are ignored and switching frequency carrier signal is much higher than the fundamental frequency.

From the figure 3 an analysis of uv phase is conducted. Voltage equation uv (v_{uv}) can be written as follows:

$$v_{uv} = e_{uv} + R_L i_{uv} + L_L \frac{di_{uv}}{dt} \quad (1)$$

Description: v_{uv} = voltage output phases,
 i_{uv} = load current

Load current i_{uv} consists of the average and harmonic component, it can be written as follows:

$$i_{uv} = \bar{i}_{uv} + \tilde{i}_{uv} \quad (2)$$

Upon substituting eqn. (2) into eqn. (1) the following equation is obtained as

$$v_{uv} = R_L \tilde{i}_{uv} + L_L \frac{d\tilde{i}_{uv}}{dt} + \left(e_{uv} + R_L \bar{i}_{uv} + L_L \frac{d\bar{i}_{uv}}{dt} \right) \quad (3)$$

$$v_{uv} = R_L \tilde{i}_{uv} + L_L \frac{d\tilde{i}_{uv}}{dt} + \bar{v}_{uv} \quad (4)$$

Component \bar{v}_{uv} is the average voltage component in phase to phase voltage uv. When the harmonic component of current is small, the voltage drop across the resistance due to the harmonic component can be neglected. The eqn. 4 becomes:

$$\tilde{i}_{uv} = \int \frac{v_{uv} - \bar{v}_{uv}}{L_L} dt \quad (5)$$

By analyzing the voltage signal vuv in the figure 3(c) using eqn. (5), obtained:

$$\tilde{i}_{uv} = \frac{E_d}{L_L} \begin{cases} -(t - t_0) & , t_0 \leq t < t_1 \\ -T_0 + \left(\frac{t-t_0}{\bar{v}_{uv}} - 1 \right) (t - t_1) & , t_1 \leq t < t_2 \\ -(t - t_2) & , t_2 \leq t < t_3 \\ -T_2 + \left(\frac{t-t_2}{\bar{v}_{uv}} - 1 \right) (t - t_3) & , t_3 \leq t < t_4 \\ -(t - t_4) & , t_4 \leq t < t_5 \end{cases} \quad (6)$$

$$\text{with } \bar{v}_{uv} = \frac{T_1 + T_2}{T_0 + T_1 + T_2 + T_3} E_d \quad (7)$$

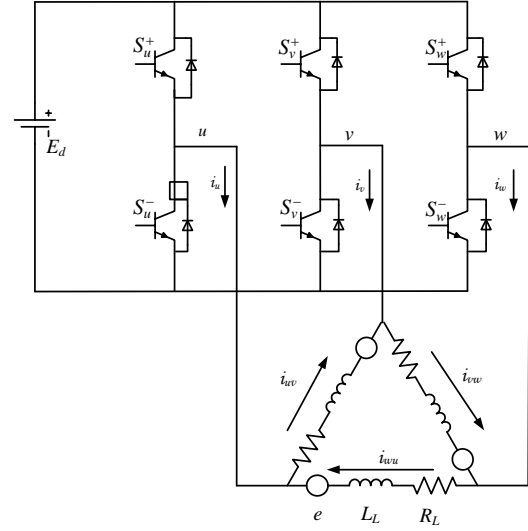


Figure 1. Scheme of three-phase PWM inverter

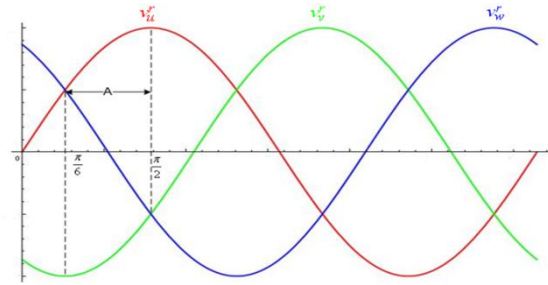


Figure 2. Three-phase reference signal

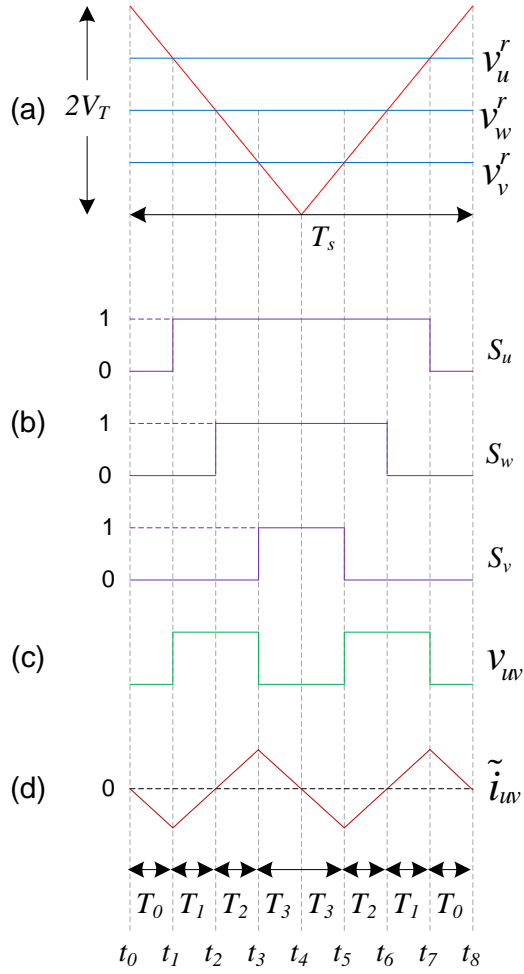


Figure 3. PWM wave in one switching period (a) the reference signal and carrier signal (b) switching state of each phase (c) output voltage v_{uv} (d) current ripple i_{uv}

It should be noted that eqn. (6) is valid during period of $-\frac{\pi}{6}$ to $\frac{5\pi}{6}$ when the reference signal

v_u^r is greater than the reference signal v_v^r .

Time intervals T_0, T_1, T_2, T_3 in relation to the carrier period (T_s) can be written as

$$\frac{2T_0}{T_s} = \frac{1}{2} - \frac{1}{2} \frac{v_u^r}{V_T} \quad (8)$$

$$\frac{2T_1}{T_s} = \frac{1}{2} \frac{v_u^r - v_w^r}{V_T} \quad (9)$$

$$\frac{2T_2}{T_s} = \frac{1}{2} \frac{v_w^r - v_v^r}{V_T} \quad (10)$$

$$\frac{2T_3}{T_s} = \frac{1}{2} + \frac{1}{2} \frac{v_v^r}{V_T} \quad (11)$$

V_T is the amplitude of the triangular carrier signal. When normalized with respect to V_T , reference signals can be written as

$$\frac{v_u^r}{V_T} = k \sin \theta + s_0 \quad (12)$$

$$\frac{v_w^r}{V_T} = k \sin \left(\theta - \frac{2\pi}{3} \right) + s_0 \quad (13)$$

$$\frac{v_v^r}{V_T} = k \sin \left(\theta + \frac{2\pi}{3} \right) + s_0 \quad (14)$$

With $\theta = 2\pi f_r t$, f_r is the frequency modulation signal, k is the modulation index and signal s_0 is injected into the reference signal in equation (12) - (14). Injection signal s_0 is added to reduce the harmonics that occur. Symmetrical and asymmetrical discontinuous PWM injection signal is used.

The mean square value of the current ripple over one switching period can be obtained by integrating the square value of eqn. (6) over the period of $t_0 - t_8$ (see figure 3). Because the current ripple waveform symmetric with respect to the point at $t = t_4$, the integration can be done over the half switching period then multiplied by two.

$$\tilde{I}_{uv}^2 = \frac{1}{T_s} \int_{t_0}^{t_0+T_s} i_{uv}^2 dt = \frac{2}{T_s} \int_{t_0}^{t_4} i_{uv}^2 dt \quad (15)$$

$$\tilde{I}_{uv}^2 = \frac{2v_{uv}^2}{T_s L_L^2} \left\{ \int_0^{T_0} t^2 dt + \int_0^{T_1+T_2} \left(-T_0 + \left(\frac{E_d}{v_{uv}} - 1 \right) (t - t_1) \right)^2 dt + \int_0^{T_3} t^2 dt \right\} \quad (16)$$

$$\tilde{I}_{uv}^2 = \frac{2v_{uv}^{-2}}{T_s L_L^2} \left\{ \frac{T_0^2}{2} + T_0^2 (T_1 + T_2) + \left(\frac{E_d}{v_{uv}} - 1 \right)^2 \frac{(T_1 + T_2)^3}{3} - \left(\frac{E_d}{v_{uv}} - 1 \right) T_0 (T_1 + T_2)^2 + \frac{T_3^2}{2} \right\} \quad (17)$$

Substituting equation (12) - (14) into equation (8) - (11). The results then incorporated into the equation (17). The quadratic equation for each output current ripple phases over a switching period is obtained as follows:

$$\tilde{I}_{uv}^2 = \frac{E_d^2}{64L_L^2 f_s^2} k^2 \sin^2 \left(\theta + \frac{\pi}{6} \right) \left[\frac{1 - \sqrt{3}k \sin \left(\theta + \frac{\pi}{6} \right) + \frac{3}{4}k^2}{+3s_0^2 + 3ks_0 \sin \left(\theta - \frac{\pi}{3} \right)} \right] \quad (18)$$

Furthermore, to find the rms current ripple over one fundamental period, the equation (18) is used. Equation to find the rms value of current ripple is given as follows:

$$\tilde{I}_{uv,rms} = \left[\frac{1}{\pi} \int_{-\frac{\pi}{6}}^{\frac{5\pi}{6}} \tilde{I}_{uv}^2 d\theta \right]^{\frac{1}{2}} \quad (19)$$

Average voltage of each phase will be the same as it gets the same injection signal, then we can use only one equation to find the rms current ripple over one fundamental period.

In general, PWM discontinuous injection signal s_0 at maximum is 1 - maximum phase or at minimum time -1 - minimum phase. Reference signal for each discontinuous technique can be seen in Table 1-3. Each reference signal's shape DPWM technique can be seen in Figure 4-6. Function so that the signal injection is used to generate modulation signal can be derived from a table with equation as follows:

$$s_0 = v_{n,discontinu}^r - \frac{v_n^r}{V_T} \quad (20)$$

With $n = u, v, w$.

Inverter output current ripple equation using discontinuous modulation can be derived by eqn. (18) and eqn. (19) using the desired injection signal, symmetrical or asymmetrical discontinuous PWM.

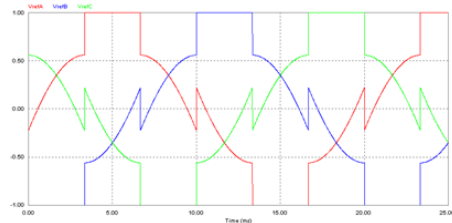


Figure 4. The symmetrical discontinuous modulation signal

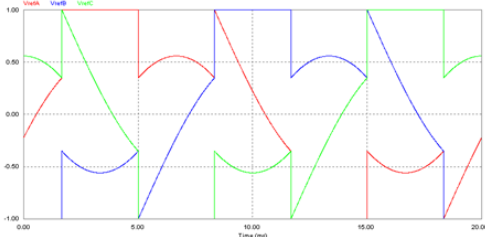


Figure 5. The asymmetrical discontinuous modulation signals with phase shift lead 30°

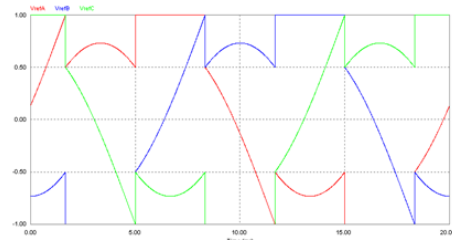


Figure 6. The asymmetrical discontinuous modulation signals with phase shift lag 30°

Table 1. Reference voltage equation for each phase on symmetrical discontinuous modulation

fasa	$v_{u,discontinu}^r$	$v_{v,discontinu}^r$	$v_{w,discontinu}^r$
$0 - \frac{\pi}{3}$	$-1 + v_{uv}$	-1	$-1 - v_{vw}$
$\frac{\pi}{3} - \frac{2}{3}\pi$	1	$1 - v_{uv}$	$1 + v_{wu}$
$\frac{2}{3}\pi - \pi$	$-1 - v_{wu}$	$-1 + v_{vw}$	-1
$\pi - \frac{4}{3}\pi$	$1 + v_{uv}$	1	$-1 - v_{vw}$
$\frac{4}{3}\pi - \frac{5}{3}\pi$	-1	$-1 - v_{uv}$	$-1 + v_{wu}$
$\frac{5}{3}\pi - 2\pi$	$1 - v_{wu}$	$1 + v_{vw}$	1

Table 2. Reference voltage equation for each phase on discontinuous asymmetric modulation with the phase shift lead 30°

fasa	$v_{u,discontinu}^r$	$v_{v,discontinu}^r$	$v_{w,discontinu}^r$
$-\frac{\pi}{6} - \frac{\pi}{6}$	$-1 + v_{uv}$	-1	$-1 - v_{vw}$
$\frac{\pi}{6} - \frac{\pi}{2}$	1	$1 - v_{uv}$	$1 + v_{wu}$
$\frac{\pi}{2} - \frac{5}{6}\pi$	$-1 - v_{wu}$	$-1 + v_{vw}$	-1
$\frac{5}{6}\pi - \frac{7}{6}\pi$	$1 + v_{uv}$	1	$-1 - v_{vw}$
$\frac{7}{6}\pi - \frac{3}{2}\pi$	-1	$-1 - v_{uv}$	$-1 + v_{wu}$
$\frac{3}{2}\pi - \frac{11}{6}\pi$	$1 - v_{wu}$	$1 + v_{vw}$	1

Table 3. Reference voltage equation for each phase on discontinuous asymmetric modulation with the phase shift lag 30°

fasa	$v_{u,discontinu}^r$	$v_{v,discontinu}^r$	$v_{w,discontinu}^r$
$-\frac{\pi}{6} - \frac{\pi}{6}$	$1 - v_{wu}$	$1 + v_{vw}$	1
$\frac{\pi}{6} - \frac{\pi}{2}$	$-1 + v_{uv}$	-1	$-1 - v_{vw}$
$\frac{\pi}{2} - \frac{5}{6}\pi$	1	$1 - v_{uv}$	$1 + v_{wu}$
$\frac{5}{6}\pi - \frac{7}{6}\pi$	$-1 - v_{wu}$	$-1 + v_{vw}$	-1
$\frac{7}{6}\pi - \frac{3}{2}\pi$	$1 + v_{uv}$	1	$-1 - v_{vw}$
$\frac{3}{2}\pi - \frac{11}{6}\pi$	-1	$-1 - v_{uv}$	$-1 + v_{wu}$

The equation for injection signal and rms ripple current equation obtained as

- Symmetrical DPWM injection signal

$$s_0 = \begin{cases} -1 - k\sin\left[\theta - \frac{2\pi}{3}\right] & , 0 \leq \theta < \frac{\pi}{3} \\ 1 - k\sin[\theta] & , \frac{\pi}{3} \leq \theta < \frac{2\pi}{3} \\ -1 - k\sin\left[\theta + \frac{2\pi}{3}\right] & , \frac{2\pi}{3} \leq \theta < \pi \\ 1 - k\sin\left[\theta - \frac{2\pi}{3}\right] & , \pi \leq \theta < \frac{4\pi}{3} \\ -1 - k\sin[\theta] & , \frac{4\pi}{3} \leq \theta < \frac{5\pi}{3} \\ 1 - k\sin\left[\theta + \frac{2\pi}{3}\right] & , \frac{5\pi}{3} \leq \theta < 2\pi \end{cases} \quad (21)$$

$$\bar{i}_{rms} = \frac{E_d k}{192\pi f_s L_1} \sqrt{6\pi(-16(45 + 8\sqrt{3})k + 192\pi + 27k^2(\sqrt{3} + 4\pi))} \quad (22)$$

- Asymmetrical DPWM injection signal shifted lead 30°

$$s_0 = \begin{cases} -1 - k\sin\left[\theta - \frac{2\pi}{3}\right] & , -\frac{\pi}{6} \leq \theta < \frac{\pi}{6} \\ 1 - k\sin[\theta] & , \frac{\pi}{6} \leq \theta < \frac{\pi}{2} \\ -1 - k\sin\left[\theta + \frac{2\pi}{3}\right] & , \frac{\pi}{2} \leq \theta < \frac{5\pi}{6} \\ 1 - k\sin\left[\theta - \frac{2\pi}{3}\right] & , \frac{5\pi}{6} \leq \theta < \frac{7\pi}{6} \\ -1 - k\sin[\theta] & , \frac{7\pi}{6} \leq \theta < \frac{3\pi}{2} \\ 1 - k\sin\left[\theta + \frac{2\pi}{3}\right] & , \frac{3\pi}{2} \leq \theta < \frac{11\pi}{6} \end{cases} \quad (23)$$

- Asymmetrical DPWM injection signal shifted lag 30°

$$s_0 = \begin{cases} 1 - k\sin\left[\theta + \frac{2\pi}{3}\right] & , -\frac{\pi}{6} \leq \theta < \frac{\pi}{6} \\ -1 - k\sin\left[\theta - \frac{2\pi}{3}\right] & , \frac{\pi}{6} \leq \theta < \frac{\pi}{2} \\ 1 - k\sin[\theta] & , \frac{\pi}{2} \leq \theta < \frac{5\pi}{6} \\ -1 - k\sin\left[\theta + \frac{2\pi}{3}\right] & , \frac{5\pi}{6} \leq \theta < \frac{7\pi}{6} \\ 1 - k\sin\left[\theta - \frac{2\pi}{3}\right] & , \frac{7\pi}{6} \leq \theta < \frac{3\pi}{2} \\ -1 - k\sin[\theta] & , \frac{3\pi}{2} \leq \theta < \frac{11\pi}{6} \end{cases} \quad (24)$$

Both type of asymmetrical DPWM have the same rms output current ripple as follows:

$$\bar{i}_{rms} = \frac{E_d k}{192\pi f_s L_1} \sqrt{3\pi(\sqrt{3}k(-1120 + 81k) + 24(16 + 9k^2)\pi)} \quad (25)$$

Maximum modulation index obtained from symmetrical and asymmetrical discontinuous PWM is as follows:

$$k_{maks} = \frac{1}{\sin\left[\frac{\pi}{3}\right]} = 1,1547$$

As comparison, equation for rms output current ripple three-phase PWM inverter with SPWM can be derived by using equation (18) and

(19) by including the value of s_0 (injection signal) variable equal to 0. Equation for rms output current ripple with SPWM obtained as follows:

$$\bar{i}_{rms} = \frac{E_d k}{f_s L_1} \sqrt{6\pi(-32\sqrt{3}k + 12\pi + 9k^2\pi)} \quad (26)$$

The result can be seen on figure 7.

3. Influence of Phase Shift on the Output Current Ripple of Three-phase PWM Inverter

Phase shift is done to find where discontinuous position produce minimum current ripple at the output of the inverter. Equation for injection signal with the phase shift is as follows:

$$s_0 = \begin{cases} -1 - k\sin\left[\theta + \alpha - \frac{2\pi}{3}\right] & , \alpha \leq \theta < \frac{\pi}{3} + \alpha \\ 1 - k\sin[\theta + \alpha] & , \frac{\pi}{3} + \alpha \leq \theta < \frac{2\pi}{3} + \alpha \\ -1 - k\sin\left[\theta + \alpha + \frac{2\pi}{3}\right] & , \frac{2\pi}{3} + \alpha \leq \theta < \pi + \alpha \\ 1 - k\sin\left[\theta + \alpha - \frac{2\pi}{3}\right] & , \pi + \alpha \leq \theta < \frac{4\pi}{3} + \alpha \\ -1 - k\sin[\theta + \alpha] & , \frac{4\pi}{3} + \alpha \leq \theta < \frac{5\pi}{3} + \alpha \\ 1 - k\sin\left[\theta + \alpha + \frac{2\pi}{3}\right] & , \frac{5\pi}{3} + \alpha \leq \theta < 2\pi + \alpha \end{cases} \quad (27)$$

In the same way by finding equation for DPWM symmetrical and asymmetrical, the rms output current ripple equation is derived as

$$\begin{aligned} \bar{i}_{rms} &= \frac{E_d k}{192\pi f_s L_1} \sqrt{3\pi(8(M) - 27k(\sqrt{3}(32+k)\cos[\alpha] - 4\sqrt{3}\cos[2\alpha] + (N)))} \\ M &= -32\sqrt{3}k + 48\pi + 27k^2\pi \\ N &= -32 + 3k + 24k\cos[\alpha]\sin[\alpha] \end{aligned} \quad (28)$$

In the same way as has been described previously, the maximum modulation index obtained as

$$k_{maks} = \frac{1}{\sin\left[\frac{\pi}{3} + \alpha\right]\cos[\alpha]} \quad (29)$$

Graph of the maximum modulation index (k_{maks}) as a function of phase shift α can be seen on figure 8. 3-dimensional plot of the relationship between modulation index (k), the shift angle (α), and rms output current ripple can be seen on figure 9.

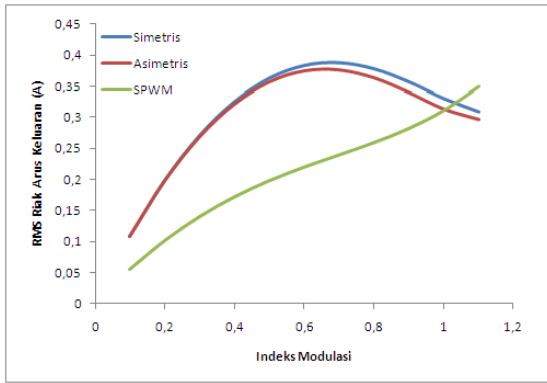


Figure 7. RMS ripple current graph of DPWM symmetrical, asymmetrical and SPWM as a function of modulation index based on analysis

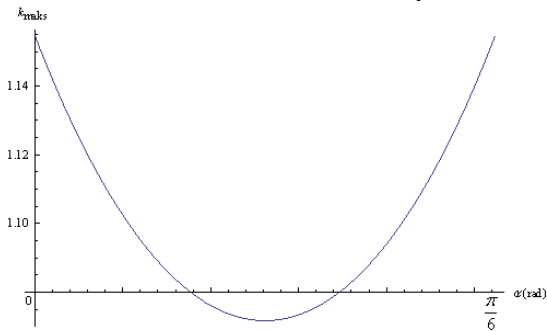


Figure 8. Graph of the maximum modulation index (k_{maks}) as a function of phase shift α

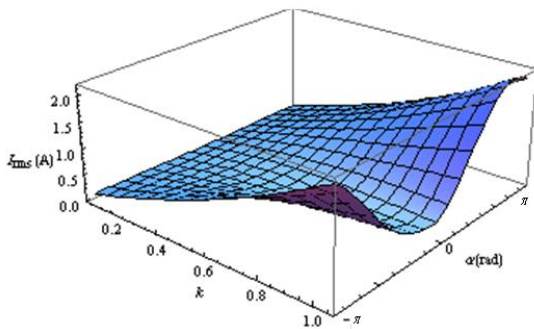


Figure 9. 3-dimensional plot of the relationship between modulation index (k), the shift angle (α), and rms output current ripple

4. Simulation Results

The analysis result proved by simulation. Plot on figure 10 - 12 illustrate the output current ripple three-phase inverter as a function of modulation index.

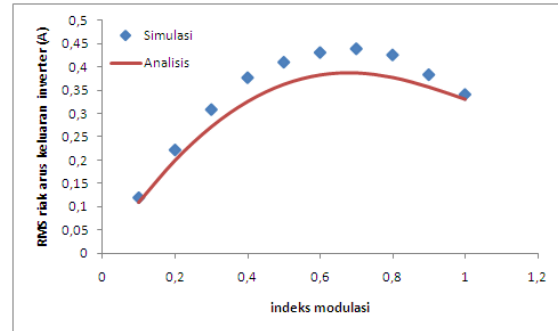


Figure 10. Graph of RMS output current ripple with symmetrical DPWM as a function of modulation index

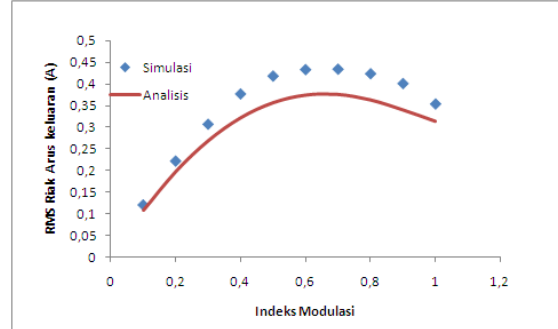


Figure 11. Graph of RMS output current ripple with asymmetrical DPWM shifted lead 30° as a function of modulation index

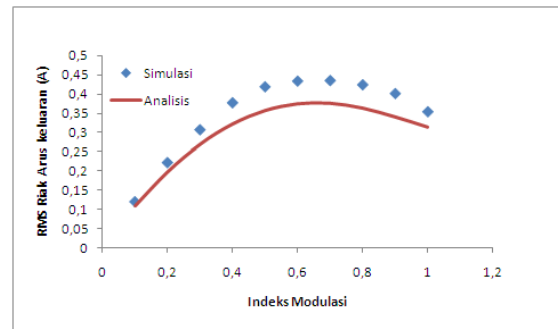


Figure 12. Graph of RMS output current ripple with asymmetrical DPWM shifted lag 30° as a function of modulation index

5. Conclusions

From analysis that conducted and the simulation results that support the analytical methods used, several conclusions can be drawn:

- Output current ripple equations of three phase inverter with discontinuous modulation has been derived and simulation results support the analysis.
- Asymmetrical DPWM produces lower current ripple than symmetrical DPWM.
- Current ripple with discontinuous modulation is influenced by variable phase shift α on injection signal. Phase shift that produces the lowest

current ripple value is to shift as far $-\pi / 6$ or at lag 30° .

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